

Consider the geometric sequence with $a_3 = -16$ and $a_6 = 54$.

SCORE: ____ / 9 PTS

[a] Find the formula for a_n .

$$\begin{aligned} a_3 &= a_1 r^2 = -16 \\ a_6 &= a_1 r^5 = 54 \end{aligned} \quad \left. \vphantom{\begin{aligned} a_3 \\ a_6 \end{aligned}} \right\} \div$$

$$r^3 = -\frac{54}{16} = -\frac{27}{8}$$

$$r = -\frac{3}{2}$$

$$a_1 \left(\frac{9}{4}\right) = -16$$

$$a_1 = -\frac{64}{9}$$

$$a_n = -\frac{64}{9} \left(-\frac{3}{2}\right)^{n-1}$$

[b] 24 is a term in the sequence. Find the sum of all terms in the sequence up to and including that term.

$$-\frac{64}{9} \left(-\frac{3}{2}\right)^{n-1} = 24$$

$$\left(-\frac{3}{2}\right)^{n-1} = 24 \cdot \frac{-9}{64} = -\frac{27}{8}$$

$$n-1 = 3$$

$$n = 4$$

$$\begin{aligned} S_4 &= \frac{-\frac{64}{9} \left(1 - \left(-\frac{3}{2}\right)^4\right)}{1 - \left(-\frac{3}{2}\right)} \\ &= \frac{-\frac{64}{9} \left(1 - \left(\frac{3}{2}\right)^4\right)}{\frac{5}{2}} \\ &= \frac{2}{5} \cdot -\frac{64}{9} \left(1 - \left(\frac{3}{2}\right)^4\right) = \frac{-128}{45} \left(1 - \left(\frac{3}{2}\right)^4\right) \\ &\approx 11.5556 \end{aligned}$$

Write $\frac{31}{2} - \frac{25}{6} + \frac{19}{24} - \frac{13}{120} + \frac{7}{720} - \frac{1}{5040}$ using sigma notation.

NUMERATORS: ARITHMETIC $d = -6$

DENOMINATORS: FACTORIALS

SCORE: ____ / 4 PTS

$$\sum_{n=1}^6 (-1)^{n+1} \frac{31-6(n-1)}{(n+1)!} = \sum_{n=1}^6 (-1)^{n+1} \frac{37-6n}{(n+1)!}$$

Handwritten annotations: Red circles containing $\frac{1}{2}$ are placed around the summation index $n=1$, the exponent $n+1$, and the denominator $(n+1)!$. A red bracket above the summation indicates the upper limit 6 . A red circle containing 2 is placed above the numerator $31-6(n-1)$.

Find the 4th term of the sequence defined recursively by $a_1 = -1$, $a_2 = 2$, $a_n = 2a_{n-2} - na_{n-1}$ for $n \geq 3$. SCORE: ____ / 4 PTS

$$a_3 = \underline{2a_1 - 3a_2} = 2(-1) - 3(2) = \underline{-8}$$

$$a_4 = \underline{2a_2 - 4a_3} = 2(2) - 4(-8) = \underline{36}$$

① EACH

Find the value of $\sum_{i=2}^5 (-1)^{i+1} (i! - 3i^2)$.

SCORE: _____ / 5 PTS

$$= -(2-12) + (6-27) - (24-48) + (120-75)$$

$$= \underbrace{10}_{\text{red}} - \underbrace{21}_{\text{red}} + \underbrace{24}_{\text{red}} + \underbrace{45}_{\text{red}}$$

$$= \underbrace{58}_{\text{red}}$$

① EACH

Simplify $\frac{(4k-2)!}{(4k-4)!}$.

SCORE: ____ / 3 PTS

$$\frac{(4k-2)(4k-3)\cancel{(4k-4)!}}{\cancel{(4k-4)!}} = \underbrace{(4k-2)(4k-3)}_{(1)}$$

(2)

Find the sum of the first 85 terms of the sequence $\frac{11}{6} + \frac{7}{4} + \frac{5}{3} + \frac{19}{12} + \dots$

SCORE: ____ / 5 PTS

ARITHMETIC: $d = \frac{7}{4} - \frac{11}{6} = \frac{21}{12} - \frac{22}{12} = -\frac{1}{12}$ ①

CHECK: $\frac{7}{4} - \frac{1}{12} = \frac{21}{12} - \frac{1}{12} = \frac{20}{12} = \frac{5}{3}$

$$\frac{5}{3} - \frac{1}{12} = \frac{20}{12} - \frac{1}{12} = \frac{19}{12}$$

$$S_{85} = \frac{85}{2} \left(2 \cdot \frac{11}{6} + (85-1) \left(-\frac{1}{12} \right) \right) = \frac{85}{2} \left(\frac{11}{3} - 7 \right) = \frac{85}{2} \left(-\frac{10}{3} \right) = -\frac{425}{3} \approx -141.6667$$

EITHER VERSION OK ①

③