Consider the geometric sequence with $a_3 = -16$ and $a_6 = 54$.

SCORE: ____ / 9 PTS

[a] Find the formula for a_n.

$a_3 = a_1 r^2 = -16$

$a_6 = a_1 r^5 = 54$ (1) ÷

$r^3 = -\frac{54}{16} = -\frac{27}{8}$

$r = -\frac{3}{2}$ (1)

$a_1\left(\frac{9}{4}\right) = -16$

$a_1 = -\frac{64}{9}$ ($\frac{1}{2}$)

$a_n = -\frac{64}{9}\left(-\frac{3}{2}\right)^{n-1}$ (2)

[b] 24 is a term in the sequence. Find the sum of all terms in the sequence up to and including that term.

$-\frac{64}{9}\left(-\frac{3}{2}\right)^{n-1} = 24$ (1)

$\left(-\frac{3}{2}\right)^{n-1} = 24 \cdot -\frac{9}{64_8} = -\frac{27}{8}$

$n-1 = 3$

$n = 4$ ($\frac{1}{2}$)

(2)

$S_4 = \frac{-\frac{64}{9}\left(1-\left(-\frac{3}{2}\right)^4\right)}{1-\left(-\frac{3}{2}\right)} = \frac{-\frac{64}{9}\left(1-\left(\frac{3}{2}\right)^4\right)}{\frac{5}{2}}$

$= \frac{2}{5} \cdot -\frac{64}{9}\left(1-\left(\frac{3}{2}\right)^4\right) = -\frac{128}{45}\left(1-\left(\frac{3}{2}\right)^4\right)$

≈ 11.5556 (1)

Write $\frac{31}{2}-\frac{25}{6}+\frac{19}{24}-\frac{13}{120}+\frac{7}{720}-\frac{1}{5040}$ using sigma notation.

NUMERATORS: ARITHMETIC $d=-6$

DENOMINATORS: FACTORIALS

SCORE: _____ / 4 PTS

$$\sum_{n=1}^{6}(-1)^{n+1}\frac{31-6(n-1)}{(n+1)!}=\sum_{n=1}^{6}(-1)^{n+1}\frac{37-6n}{(n+1)!}$$

Find the 4^{th} term of the sequence defined recursively by $a_1 = -1, \quad a_2 = 2, \quad a_n = 2a_{n-2} - na_{n-1}$ for $n \geq 3$. **SCORE: ____ / 4 PTS**

$a_3 = 2a_1 - 3a_2 = 2(-1) - 3(2) = -8$

$a_4 = 2a_2 - 4a_3 = 2(2) - 4(-8) = 36$

(1) EACH

Find the value of $\sum_{i=2}^{5}(-1)^{i+1}(i!-3i^2)$.

SCORE: _____ / 5 PTS

$= -(2-12)+(6-27)-(24-48)+(120-75)$

$= 10-21+24+45$

$= 58$

(1) EACH

Simplify $\frac{(4k-2)!}{(4k-4)!}$.

SCORE: _____ / 3 PTS

$$\frac{(4k-2)(4k-3)(4k-4)!}{(4k-4)!} = (4k-2)(4k-3)$$

(2) (1)

Find the sum of the first 85 terms of the sequence $\frac{11}{6}+\frac{7}{4}+\frac{5}{3}+\frac{19}{12}+\cdots$.

SCORE: ____ / 5 PTS

ARITHMETIC: $d = \frac{7}{4} - \frac{11}{6} = \frac{21}{12} - \frac{22}{12} = -\frac{1}{12}$ (1)

CHECK: $\frac{7}{4} - \frac{1}{12} = \frac{21}{12} - \frac{1}{12} = \frac{20}{12} = \frac{5}{3}$

$\frac{5}{3} - \frac{1}{12} = \frac{20}{12} - \frac{1}{12} = \frac{19}{12}$

$$S_{85} = \frac{85}{2}\left(2\cdot\frac{11}{6} + (85-1)\left(-\frac{1}{12}\right)\right) = \frac{85}{2}\left(\frac{11}{3} - 7\right) = \frac{85}{2}\left(-\frac{10}{3}\right) = -\frac{425}{3} \approx -141.6667$$

(3)

EITHER VERSION OK (1)